A Multiframe Assignment Algorithm for Single Sensor Bearings-Only Tracking

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Abstract – *Bearings-only tracking (BOT) using a single maneuvering platform has been studied extensively in the past. However, only a few studies exist in the open literature that deal with measurement origin uncertainty. Most publications are concerned with finding the best filtering approach, since BOT is inherently nonlinear, or finding the optimal maneuver strategy for the sensor platform to improve observability. We consider measurement origin uncertainty due to the existence of multiple targets in the surveillance region, non-unity detection probability, and false alarms. Our algorithm uses the multiframe assignment (MFA) to solve the data association problem, and filtering is performed using the unscented Kalman filter (UKF). We employ both the modified and log polar coordinate systems. Simulation results show that the proposed algorithm is very effective in terms of tracking accuracy and track maintenance capability, especially when formulated in the log polar coordinate system.*

Keywords: Bearings-only tracking, multiframe assignment, data association, unscented Kalman filter, modified polar coordinates, log polar coordinates.

1 Introduction

The problem of bearings-only tracking (BOT) has several important practical applications such as submarine tracking using passive sonar or aircraft surveillance using a radar in passive mode [18]. The objective of BOT is to estimate the target state consisting of kinematic parameters such as position and velocity from the measured line-of-sight angle-of-arrival (bearing) of the received signal. Since the range to the target is not observable from a single bearing measurement, to satisfy observability of the complete target state, a BOT system must employ multiple spatially separated sensors. If, however, a single sensor is used, then the sensor platform (own-ship) should maneuver at least one degree higher than the maximum possible target maneuver [22].

Measurements from different sensors in a multisensor BOT tracking system can be synchronous or asynchronous. In the case of synchronous sensors, to track multiple targets a two-step assignment algorithm was proposed in [15][17], where first the bearing measurements from different sensors are associated using a multidimensional assignment, and then a two dimensional assignment is used to associate the resulting composite measurements to the tracks. In [21] a computationally efficient, single step, assignment-based algorithm was proposed for multiple synchronous sensors. The multisensor asynchronous BOT was the considered in several publications including [8][14][20].

There are two important issues that need to be addressed in BOT using a single sensor: the inherent nonlinearity of the filtering problem and the observability of the target. Most publications have tried to address these issues for a single non-maneuvering target and have proposed either batch processing or recursive algorithms. Well-known recursive algorithms include the pseudo-linear filter [13], the modified polar coordinates (MPC) extended Kalman filter (EKF) [2], the range-parameterized EKF in both the modified [16] and Cartesian [12] coordinate systems.

Only a few publications have considered BOT of maneuvering targets with measurement origin uncertainty. The maximum likelihood probabilistic data association algorithm (ML-PDA) was proposed in [11] for a single target in clutter, and it was extended to handle multiple targets [6]. These are batch processing algorithms and, as mentioned in [6], may not be practical for three or more targets. A particle filtering algorithm was developed for multiple target tracking in [9] and was applied to multitarget BOT problem. Both [6] and [9] are applicable only to scenarios where the number of targets are known.

In this paper we consider the problem of BOT of multiple targets with measurement origin uncertainty, where the target motion is modeled using a nearly constant velocity model (NCV). The assumption that the target motion obeys an NCV model is not a restrictive requirement on the part of the proposed algorithm, rather it is due to the complexity of the own-ship maneuver that may be required with respect to multiple targets, if the targets can have other motion types such as coordinated turn. The important issue of data association is solved using a sliding-window multiframe assignment (MFA) algorithm [15], which is a discrete optimization formulation and an alternative to the popular probabilistic data association [3]. The MFA algorithm effectively models the multiple hypothesis tracker within the window. To track detected targets the unscented Kalman filter (UKF) is used.

The rest of this paper is organized as follows. The BOT problem is described in detail in the next section. The proposed algorithm is presented in Section 3. The MFA algorithm for data association is described in Section 4. Results of the simulation study conducted are presented in Section 5, followed by concluding remarks in Section 6.

2 Problem Description

We consider the problem of tracking multiple targets using a single bearings-only sensor mounted to a maneuvering platform in the two-dimensional plane. The state of target i at scan k corresponding to time t_k is denoted by $\mathbf{x}_{k,i}$ and it consists of the position and velocity components in the x and y Cartesian directions, i.e., $\mathbf{x}_{k,i} = [x_{k,i}, \dot{x}_{k,i}, y_{k,i}, \dot{y}_{k,i}]^T$. Similarly the state of the observer is defined as $\mathbf{x}_{k,o} = [x_{k,o}, \dot{x}_{k,o}, y_{k,o}, \dot{y}_{k,o}]^T$ and is assumed to be known for all values of k.

Define the relative state of target i as $\mathbf{x}^r_{k,i} = \mathbf{x}_{k,i} - \mathbf{x}_{k,o}$, then the discrete-time state transition equation can be written as [18]

$$\mathbf{x}^r_{k,i} = \mathrm{F}_{k,k-1}\mathbf{x}^r_{k-1,i} + \mathbf{v}_{k-1} - \mathrm{U}_{k,k-1} \tag{1}$$

where $\mathrm{F}_{k,k-1}$ is the state transition matrix, $\mathbf{v}_{k-1}$ is a zero-mean white Gaussian noise sequence with covariance matrix Q_{k-1} that models the maneuvers of the target, and $\mathrm{U}_{k,k-1}$ is a deterministic control input that accounts for the acceleration of the observer. $\mathrm{F}_{k,k-1}$ and Q_k are given by

$$\mathrm{F}_{k,k-1} = \mathrm{I}_2 \otimes \begin{bmatrix} 1 & T_k \\ 0 & 1 \end{bmatrix} \tag{2}$$

$$\mathrm{Q}_k = \mathrm{I}_2 \otimes \begin{bmatrix} T_k^3/3 & T_k^2/2 \\ T_k^2/2 & T_k \end{bmatrix} \tilde{q} \tag{3}$$

where $T_k = t_k - t_{k-1}$, $\tilde{q}$ is the intensity of the noise sequence v_k, I_2 is a 2×2 identity matrix, and $\otimes$ denotes the Kronecker product. $\mathrm{U}_{k,k-1}$ is given by

$$\mathrm{U}_{k,k-1} = \begin{bmatrix} u_{k1} \\ u_{k2} \\ u_{k3} \\ u_{k4} \end{bmatrix} = \begin{bmatrix} x_{k,o} - x_{k-1,o} - T_k\dot{x}_{k-1,o} \\ \dot{x}_{k,o} - \dot{x}_{k-1,o} \\ y_{k,o} - y_{k-1,o} - T_k\dot{y}_{k-1,o} \\ \dot{y}_{k,o} - \dot{y}_{k-1,o} \end{bmatrix} \tag{4}$$

The sensor measures the bearing of the targets in the surveillance region with probability of detection P_D. It can also generate false detections. Assume that M_k measurements are reported by the sensor at time t_k from N_k targets (note M_k need not be equal to N_k) and that these measurements are grouped in a vector Θ_k, i.e., $\Theta_k = \{\theta_{k,j}\}_{j=1}^{M_k}$, where $\theta_{k,j}$ denote the jth measurement, which is a scalar.

We assume that the bearing measurements are measured from the y-axis with positive angles in the clockwise direction. The measurement equation can be written as

$$\theta_{k,j} = \begin{cases} h(\mathbf{x}^r_k) + w_k & \text{if target originated} \\ \tilde{\theta}_k & \text{if false alarm} \end{cases} \tag{5}$$

where w_k is the bearing measurement noise, which is assumed to be zero-mean Gaussian distributed with standard deviation σ_θ. The measurement function $h(\mathbf{x}^r)$ is given by

$$h(\mathbf{x}^r_k) = \arctan\left(\frac{x^r}{y^r}\right) \tag{6}$$

where (x^r, y^r) is the relative target position. A target index is not used in the measurement equation (5), because the association between the measurements and the targets is not known to the tracker. The false measurement $\tilde{\theta}_k$ is assumed to be uniformly distributed in the field of view of the sensor and the number of false alarms is assumed to be Poisson distributed.

The objective of BOT is to track multiple unknown number of targets using the measurements Θ_k, $k = 1, 2, \ldots$ reported by the sensor.

2.1 Alternate Coordinate Systems for BOT

Observe that the state-space model of the BOT problem has a linear state equation and a nonlinear measurement equation in the Cartesian coordinate system and hence precludes the use of conventional Kalman filter (KF). The use of EKF has been shown as producing unstable estimation behavior [1]. A well-known stable solution was to formulate the filtering problem in the MPC system [2]. A recent development along this coordinate transformation spirit is the development of log polar coordinate (LPC) system [7].

In these coordinate systems, contrary to the Cartesian coordinate system, the state equation is nonlinear and the measurement equation is linear. More importantly, the observable and unobservable components of the target state are decoupled. We tested the proposed algorithm in both these coordinate systems.

The state transition of both the MPC and LPC systems can be written

$$\mathbf{y}_{k,i} = \mathrm{f}(\mathbf{y}_{k-1,i}) \tag{7}$$

where $\mathbf{y}_{k,i}$ is the relative state vector of target i at scan k and the measurement equation is given by

$$\theta_{k,j} = \begin{cases} \mathrm{H}\mathbf{y}_k + w_k & \text{if target originated} \\ \tilde{\theta}_k & \text{if false alarm} \end{cases} \tag{8}$$

where $\mathrm{H} = [0, 0, 1, 0]$ is the measurement matrix for both MPC and LPC.

In MPC the state vector is defined as $\mathbf{y}_{k,i} = [1/r_{k,i}, \dot{r}_{k,i}/r_{k,i}, \beta_{k,i}, \dot{\beta}_{k,i}]^T$, where the components are the reciprocal of the range, the ratio between range rate and range, bearing, and bearing rate. The state transition function in MPC can be written as [2]

$$f(\mathbf{y}_k) = \begin{bmatrix} \frac{y_1}{\sqrt{S_3^2+S_4^2}} \\ \frac{S_1 S_3 + S_2 S_4}{S_3^2+S_4^2} \\ y_3 + \arctan\left(\frac{S_3}{S_4}\right) \\ \frac{S_1 S_4 - S_2 S_3}{S_3^2+S_4^2} \end{bmatrix} \tag{9}$$

where

$$S_1 = y_4 - y_1 \left(u_{k3}\cos y_3 - u_{k4}\sin y_3\right) \tag{10}$$

$$S_2 = y_2 - y_1 \left(u_{k3}\sin y_3 + u_{k4}\cos y_3\right) \tag{11}$$

$$S_3 = T_k y_4 - y_1 \left(u_{k1}\cos y_3 - u_{k2}\sin y_3\right) \tag{12}$$

$$S_4 = 1 + T_k y_2 - y_1 \left(u_{k1}\sin y_3 + u_{k2}\cos y_3\right) \tag{13}$$

In the above we have used the shorthand notation y_1 to y_4 to denote the state variables at scan $k-1$, i.e., $\mathbf{y}_{k-1} = [y_1, y_2, y_3, y_4]^T$.

The last three components of the state vector in the LPC are the same as that of the MPC, however, the first component in LPC is the logarithm of the range. That is, the state vector in the LPC is defined as $\mathbf{y}_{k,i} = [\ln(r_{k,i}), \dot{r}_{k,i}/r_{k,i}, \beta_{k,i}, \dot{\beta}_{k,i}]^T$. The state transition function is similar to the one shown in (9), except the variables S_1 to S_4 are replaced by D_1 to D_4, defined as

$$D_1 = y_4 - \exp(-y_1)\left(u_{k3}\cos y_3 - u_{k4}\sin y_3\right) \tag{14}$$

$$D_2 = y_2 - \exp(-y_1)\left(u_{k3}\sin y_3 + u_{k4}\cos y_3\right) \tag{15}$$

$$D_3 = T_k y_4 - \exp(-y_1)\left(u_{k1}\cos y_3 - u_{k2}\sin y_3\right) \tag{16}$$

$$D_4 = 1 + T_k y_2 - \exp(-y_1)\left(u_{k1}\sin y_3 + u_{k2}\cos y_3\right) \tag{17}$$

and the first element of $f(\mathbf{y}_k)$ in LPC is now given by $y_1 + 0.5\ln(D_3^2 + D_4^2)$. The conversion from either the MPC and LPC to the Cartesian coordinates and vice-versa has a nonlinear transformation.

3 Proposed Algorithm

A track is initialized for each measurement in the first scan of measurements. It is assumed that prior knowledge of initial range, speed, and course of the targets is available. The association between the subsequent scans of measurements and the tracks is found using the MFA formulation, which is described in the next section. Once the association is found, the tracks are updated using the corresponding measurements. We used the UKF to update the tracks.

3.1 Track Initialization

We use the procedure discussed in [18] to initialize tracks for each measurement in the first scan. Assume that the prior knowledge of the target range r has a normal distribution with mean $\bar{r}$ and variance σ_r^2, i.e., $r \sim \mathcal{N}(\bar{r}, \sigma_r^2)$. Similarly, the initial speed s and initial course c of the target has a normal prior distribution, i.e., $s \sim \mathcal{N}(\bar{s}, \sigma_s^2)$, and $c \sim \mathcal{N}(\bar{c}, \sigma_c^2)$. We further assume that the initial target movement is towards the own-ship.

Given these assumptions, for each bearing $\theta_{1,m}$ from the first scan, the initial target state in the Cartesian coordinates is given by

$$\mathbf{x}_{1,m}^r = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \bar{r}\sin\theta_{1,m} \\ \bar{s}\sin\bar{c} - \dot{x}_{1,o} \\ \bar{r}\cos\theta_{1,m} \\ \bar{s}\cos\bar{c} - \dot{y}_{1,o} \end{bmatrix} \tag{18}$$

Because of our assumption that the initial target motion is towards the own-ship $\bar{c} = \theta_{1,m} + \pi$. The initial state covariance matrix is given by

$$P_{1,m} = \begin{bmatrix} P_{xx} & 0 & P_{xy} & 0 \\ 0 & P_{\dot{x}\dot{x}} & 0 & P_{\dot{x}\dot{y}} \\ P_{xy} & 0 & P_{yy} & 0 \\ 0 & P_{\dot{x}\dot{y}} & 0 & P_{\dot{y}\dot{y}} \end{bmatrix} \tag{19}$$

where

$$P_{xx} = \sigma_r^2 \sin^2\theta_{1,m} + \bar{r}^2\sigma_\theta^2\cos^2\theta_{1,m} \tag{20}$$

$$P_{yy} = \sigma_r^2 \cos^2\theta_{1,m} + \bar{r}^2\sigma_\theta^2\sin^2\theta_{1,m} \tag{21}$$

$$P_{xy} = \left(\sigma_r^2 - \bar{r}^2\sigma_\theta^2\right)\sin\theta_{1,m}\cos\theta_{1,m} \tag{22}$$

$$P_{\dot{x}\dot{x}} = \sigma_s^2\sin^2\bar{c} + \bar{s}^2\sigma_c^2\cos^2\bar{c} \tag{23}$$

$$P_{\dot{y}\dot{y}} = \sigma_s^2\cos^2\bar{c} + \bar{s}^2\sigma_c^2\sin^2\bar{c} \tag{24}$$

$$P_{\dot{x}\dot{y}} = \left(\sigma_s^2 - \bar{s}^2\sigma_c^2\right)\sin\bar{c}\cos\bar{c} \tag{25}$$

The initial target state $\mathbf{x}_{1,m}^r$ is transformed to both MPC and LPC using the corresponding nonlinear transformation to initialize the filter in these coordinate systems. The linearized version of the transformation model is used to transform the covariance matrix (see [18] for details).

3.2 Filtering Algorithm

As mentioned earlier in this paper the state equation in both the MPC and LPC are nonlinear, whereas the measurement equation is linear. Therefore, the filtering problem in BOT is essentially nonlinear and the EKF has been used in many previous publications. The EKF uses a Taylor series expansion to linearize the nonlinear model, and propagates the mean and covariance of the state through the linearized state-space model. A computationally similar alternative to EKF is the UKF, which uses a deterministic sampling approach that represents the state by a carefully chosen sample points and propagates these sampling points through the nonlinear system directly without any linearization [10]. Our experience is that the performance of the UKF is almost always better than that of the EKF. Further, unlike the EKF, the UKF does not require the evaluation of the Jacobian or the Hessian of the nonlinear state function, as a result it is numerically stable compared to the EKF. Hence, we selected the UKF for filtering.

3.3 Track Maintenance

We used a simple track confirmation and deletion logic. A track is initialized for each bearing measurement in the first scan and in the subsequent scans a track is initialized for all measurements that are not associated with any tracks. This allows the algorithm to handle target births.

Tracks are classified as either confirmed and tentative based on an m out of n logic [3]. A track, when initialized, is marked tentative, and is moved to the confirmed track list, only when it is updated by m real measurements in n consecutive scans. In the simulations $m = 3$ and $n = 5$.

To reduce the chances of forming false tracks, a stringent track deletion technique was used for newly initialized tracks. If a track initialized in a particular scan is not associated with any of the measurements in the very next scan, then that track is deleted. For confirmed track deletion, however, similar to the confirmation procedure, an m out of n logic is used. In low P_D scenarios one may opt for a more relaxed track deletion approach for newly created tracks. It is also possible to use more sophisticated track score-based approach for track maintenance [5].

4 Multiframe Assignment for Data Association

The MFA algorithm associates the latest $S-1$ scans of measurements (frames) to the tracks, i.e., when the frame at scan k is received, the association is performed between the track list at scan $k-S+1$ and measurements at scans $\{k-S+s\}_{s=2}^{S}$. The MFA is often implemented as a sliding window. After performing the association at kth scan, tracks are updated only with the measurements from the $(k-S+2)$th scan. When the next frame at scan $k+1$ is received, the window is advanced to cover the frames at scans $\{k-S+s\}_{s=3}^{S+1}$ and association is performed between these frames and the tracks at $k-S+2$, and so on.

Observe that at scan k the MFA assignment algorithm makes a hard decision to only a single frame of data, i.e., after the association at scan k the tracks are updated with only the frame at scan $k-S+2$. The association between the tracks at $k-S+1$ and the rest of the frames is only tentative (i.e., soft decisions), and can be changed in the subsequent processing in light of the new measurements. This soft decision capability gives the MFA advantage over the 2-D assignment, where the tracks are associated and updated with the latest frame. The price paid for this advantage is the delayed decision making and increased computational cost.

The MFA algorithm formulates the data association as a constrained global optimization problem. The objective is to minimize the total assignment cost of associating sequences of measurements to the tracks. At scan k the cost of associating a particular sequence of measurements $\{\theta_{s,j_s}\}_{s=k-S+2}^{k}$ to a track n at scan $k-S+1$ is defined as

$$c\left(k, n, \{j_s\}_{s=k-S+2}^{k}\right) = -\ln \frac{\phi\left(k, n, \{j_s\}_{s=k-S+2}^{k}\right)}{\phi\left(k, 0, \{j_s\}_{s=k-S+2}^{k}\right)} \quad (26)$$

where $\phi\left(k, n, \{j_s\}_{s=k-S+2}^{k}\right)$ is the joint likelihood that the $(S-1)$-tuple of measurements originated from the target that is being followed by track n and $\phi\left(k, 0, \{j_s\}_{s=k-S+2}^{k}\right)$ is the likelihood that the same $S-1$ measurements are false, i.e., it corresponds to a non-existing "dummy" track. The measurement index j_s can be equal to zero as well, in which case it denotes a "dummy" measurement and is used to account for missed detections.

The association likelihoods are given by

$$\phi\left(n, \{j_s\}_{s=k'+2}^{k}\right) = \begin{cases} \prod_{s=k'+2}^{k} [1-P_D]^{1-u(j_s)} [P_D \Lambda(s,n,j_s)]^{u(j_s)} & n > 0 \\ \prod_{s=k'+2}^{k} V_s^{u(j_s)} & n = 0 \end{cases} \quad (27)$$

where $k' = k-S$, $\Lambda(s,n,j_s)$ is the filter calculated likelihood of track n when it is associated to measurement θ_{s,j_s}, V_s is the spatial false alarm density at scan s, and $u(j_s)$ is a binary indicator function defined such that $u(j_s) = 0$ for $j_s = 0$ and one otherwise.

The MFA attempts to find the best set of associations $\rho\big(n, \{m_s\}_{s=k'+2}^{k}\big)$ that minimize the global assignment cost $C(k)$ given by

$$\sum_{n=0}^{N_{k'+1}} \sum_{j_{k'+2}=0}^{M_{k'+2}} \cdots \sum_{j_k=0}^{M_k} c\left(k, n, \{j_s\}_{s=k'+2}^{k}\right) \rho\Big(n, \{m_s\}_{s=k'+2}^{k}\Big) \quad (28)$$

subject to one-to-one correspondence constraints that each measurement in a frame is assigned at most to one track, and each track is associated at most to one validated measurement in each frame. These constraints can be written as

$$\begin{aligned} &\sum_{m_{k'+2}=0}^{M_{k'+2}} \sum_{m_{k'+2}=0}^{M_{k'+3}} \cdots \sum_{m_k=0}^{M_k} \rho\Big(n, \{m_s\}_{s=k'+2}^{k}\Big) = 1, \\ &\qquad\qquad n = 1, 2, \ldots, N_{k'+1} \\ &\sum_{n=0}^{N_{k'+1}} \sum_{m_{k'+3}=0}^{M_{k'+3}} \cdots \sum_{m_{S-1}=0}^{M_k} \rho\Big(n, \{m_s\}_{s=k'+2}^{k}\Big) = 1, \\ &\qquad\qquad m_{k'+2} = 1, 2, \ldots, M_{k'+2} \\ &\qquad\qquad \vdots \\ &\sum_{n=0}^{N_{k'+1}} \sum_{m_{k'+2}=0}^{M_{k'+2}} \cdots \sum_{m_k=0}^{M_{k-1}} \rho\Big(n, \{m_s\}_{s=k'+2}^{k}\Big) = 1, \\ &\qquad\qquad m_k = 1, 2, \ldots, M_k \end{aligned} \quad (29)$$

where $\rho\big(n, \{m_s\}_{s=1}^{S-1}\big)$ is a binary variable such that

$$\rho\Big(n, \{m_s\}_{s=k'+2}^{k}\Big) = \begin{cases} 1 & \text{if track } n \text{ is assigned to } \{\theta_{s,m_s}\}_{s=k'+2}^{k} \\ 0 & \text{otherwise} \end{cases} \quad (30)$$

It can be shown that the global optimization problem defined above is NP-hard even under unity detection probability and no false alarms, when the track list is associated with two or more frames [15]. Therefore, finding the optimal solution using polynomial time complexity algorithms

Table 1: Simulation parameters

Parameter	Value
Sensor initial position	[0 0] m
Sensor velocity	[-1.97 -1.66] m/s
Target 1 initial position	[4924 868] m
Target 1 initial velocity	[-1.34 -1.13] m/s
Target 2 initial position	[4924 -868] m
Target 2 initial velocity	[-1.34 1.13] m/s
Sampling interval	1 min
P_D	0.9
σ_θ	0.5^o, 1^o
λ	2.5

is impractical, for all but very small problems. For applications such as target tracking that require real-time performance, algorithms that can give suboptimal solutions in pseudo-polynomial time are preferred.

In this paper the Lagrangian relaxation-based suboptimal algorithm [15], which can give a near optimal solution that is quantifiable through the duality gap, is used to solve the MFA problem. In this algorithm $S-2$ constraints are simultaneously relaxed using Lagrangian multipliers and the resulting 2-D assignment problem is solved optimally using a quasi-polynomial time using, for example, the auction algorithm [4]. Then the Lagrange multipliers are updated individually, which enforces the relaxed constraints. See [15] for details.

4.1 Implementation Details

We implemented the MFA as a two-step procedure. In the first step, when performing the association at scan k, only the association between the confirmed tracks at scan $k-S+1$ and the measurement at scans $\{k-S+s\}_{s=2}^{S}$ is attempted. One the association is found, all measurements in the first scan $(k-S+2)$ that are assigned to the confirmed tracks are removed from the measurement list and if the list is nonempty, then another MFA is performed between the the tentative tracks at $k-S+1$ and the updated measurement list at scan $k-S+2$, and the unaltered measurement list at scans $\{k-S+s\}_{s=3}^{S}$.

There are two reasons for considering such a two-step implementation of MFA. The first is that it complies with the notion that is prevalent in target tracking — it is preferable to assign measurements to established tracks, rather than to create new, potentially false, tracks. The second reason is that, except in worst case scenarios, such as non of the measurements at scan $k-S+2$ are assigned to the confirmed tracks, it can be shown that the two-step procedure can be computationally simpler, especially in high target density and high false alarms scenarios. This is because the number of hypotheses generated grows exponentially when a single measurement or track is added to their respective lists, and since by removing the tentative tracks from consideration in the first step and the measurements associated to the confirmed tracks in the second step, the total number of hypotheses generated by this two-step procedure is considerably lower than that is generated by a single-step procedure.

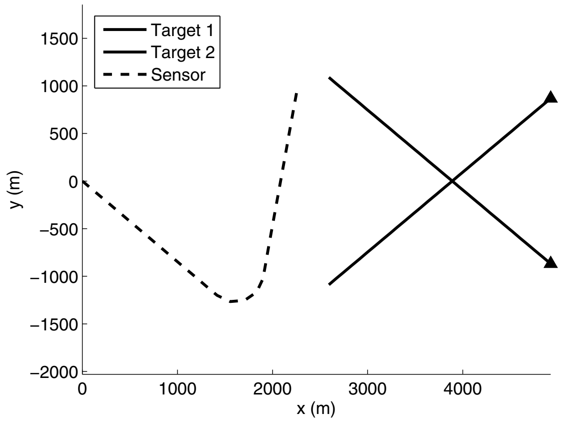

Figure 1: Sample target and own-ship trajectories. Initial locations are marked with ▲.

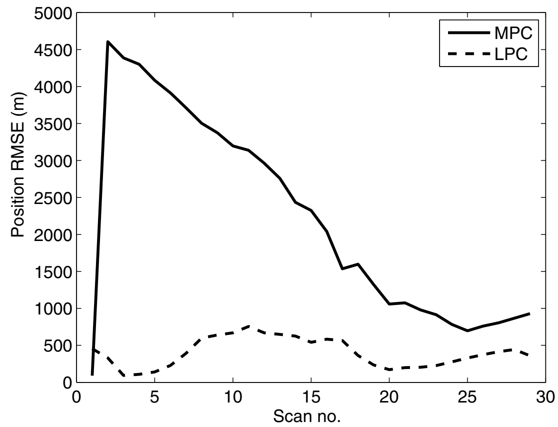

Figure 2: Position RMSE, single target tracking, $\sigma = 1^o$.

5 Simulations

We tested the proposed algorithm on a single and two target scenarios. The single target scenario is similar to the one considered in [18], and another target was added to this scenario to obtain the two target scenario.

The specific values used for various parameters are given in Table 1 and a sample trajectory of the two targets and own-ship are shown in Fig. 1. For the single target tracking only the Target 1 was considered. A nearly constant velocity model was used to generate the target trajectories. Sensor travels at constant velocity for first 13 min, then makes a 120^o coordinated turn in 4 min and then maintains a constant velocity motion for the rest of the simulation duration, which is 13 min.

The sensor generates measurements from the target with

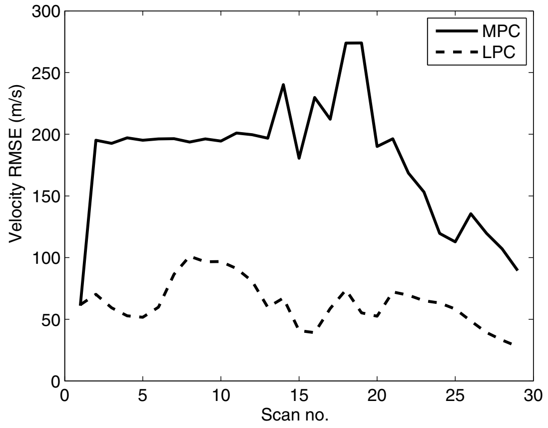

Figure 3: Velocity RMSE, single target tracking, $\sigma = 1^o$.

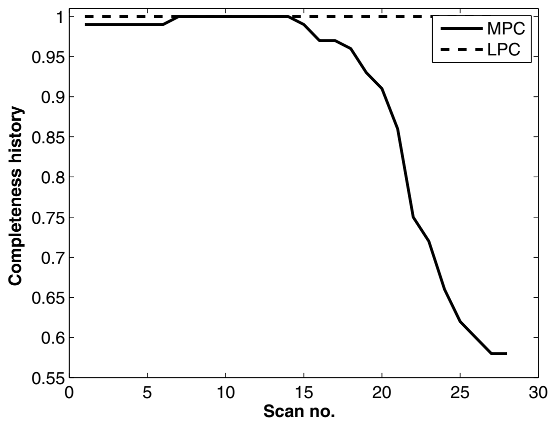

Figure 4: Completeness history, single target tracking, $\sigma = 1^o$.

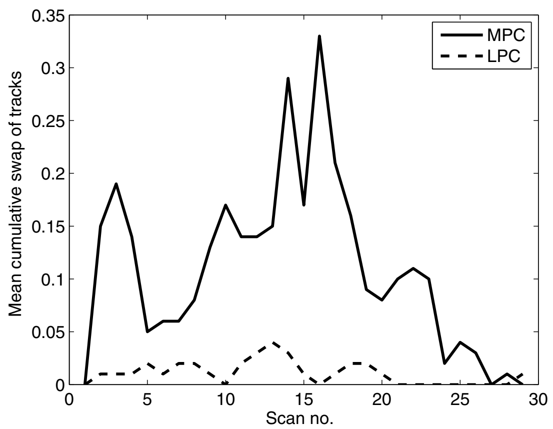

Figure 5: Mean cumulative swap of tracks, single target tracking, $\sigma = 1^o$.

the probability of detection P_D, whose measurement noise standard deviation is σ_θ. The number of false measurements are generated according to a Poisson distribution with rate λ and their values are uniformly distributed in the range $[-\pi, \pi]$ rad.

The proposed algorithm in both the MPC and LPC was run on the simulated data sets. A three frame assignment algorithm that associates the latest two scans of measurements to the tracks is used in the simulations. In both the single and two target scenarios 100 Monte Carlo runs were performed.

To evaluate the performance the root mean squared error (RMSE) of position and velocity estimates, average track length, completeness history, mean cumulative swap of tracks, and mean number of broken tracks were calculated. At each time step a unique gated assignment is carried out between the targets and the declared tracks. Based on the resulting target-track association the above mentioned metrics are calculated. Details of the calculation can be found in [19].

5.1 Results - single target scenario

The position and velocity RMSE obtained when the proposed algorithm was used to track Target 1 is shown in Fig. 2 and Fig. 3, respectively. The completeness history and mean cumulative swap of tracks are shown in Fig. 4 and Fig 5. These figures show that the performance of the proposed algorithm in the LPC is significantly better than that in the MPC. Although it seems the position and velocity RMSE of the MPC system, reduces and comes close to that of LPC once the own-ship starts maneuvering, a look at Fig. 4 shows that it held declared tracks only in 60% of the runs. Also MPC produces considerable track swap compared to the LPC as can be seen in Fig 5. We observed improved performance in both MPC and LPC when the measurement noise level was $\sigma_\theta = 0.5^o$, however, due to page limitations the results are not presented.

Table 2 shows the average track length (L_T) and the mean number of broken tracks (N_B) for this scenario. Again one can observe that the LPC tracker is considerably better compared to MPC tracker.

5.2 Results - two target scenario

The position and velocity RMSE, completeness history, and mean cumulative swap of track for the two target scenario described are shown in Fig. 6–9. The average track length and the mean number of broken tracks are shown in Table 2.

From these results it is clear that again the performance of the LPC tracker is significantly better than that of the MPC tracker. While the position and velocity RMSE of the LPC tracker has only slightly increased compared to single target scenario, the MPC tracker shows large increase in both these metrics. Such a trend is seen in other metrics presented as well. Note that in the two scenarios the mean number of broken tracks is zero for the LPC tracker, whereas that for the MPC tracker has increased from the single target to two

Table 2: Average track length and mean number of broken tracks.

		$\sigma = 0.5^o$		$\sigma = 1^o$	
		MPC	LPC	MPC	LPC
Single target case	L_T	21.0	28.1	19	26.7
	N_B	1	0	14	0
Two target case	L_T	18.6	27.1	17.3	26.5
	N_B	11	0	21	0

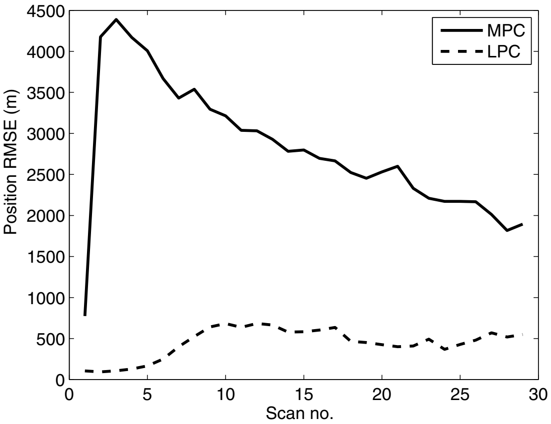

Figure 6: Position RMSE, two target scenario, $\sigma = 1^o$.

target. Also the average track length has hardly changed for the LPC tracker between these two scenarios.

6 Conclusions

In this paper we presented an algorithm for tracking multiple targets using bearings-only measurements obtained from a single maneuvering platform. The algorithm also addressed measurement origin uncertainty that arise due to non unity detection probability and false measurements. The important issue of data association was addressed using the MFA assignment algorithm, and UKF was used in the track filter using both modified and log polar coordinate systems. Our simulation results showed that the proposed algorithm was able to effectively track targets, and from the results presented one can conclude that the LPC-based tracker is the preferred approach. We are currently evaluating the algorithm in more challenging scenarios and to compare its performance against that of alternate solutions such as the joint probabilistic data association. We are also developing improved initialization techniques.

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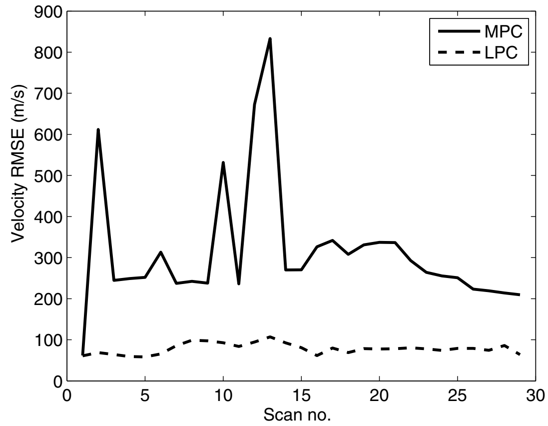

Figure 7: Velocity RMSE, two target scenario, $\sigma = 1^o$.

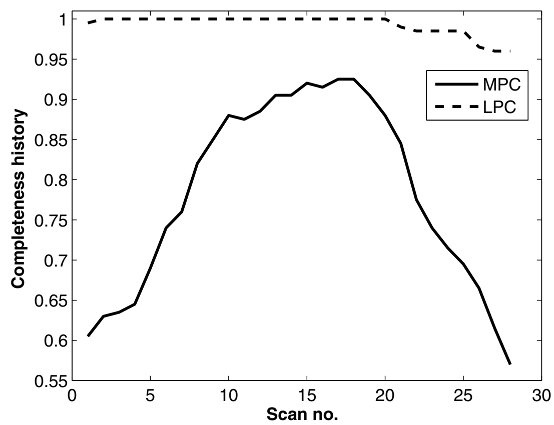

Figure 8: Completeness history, two target scenario, $\sigma = 1^o$.

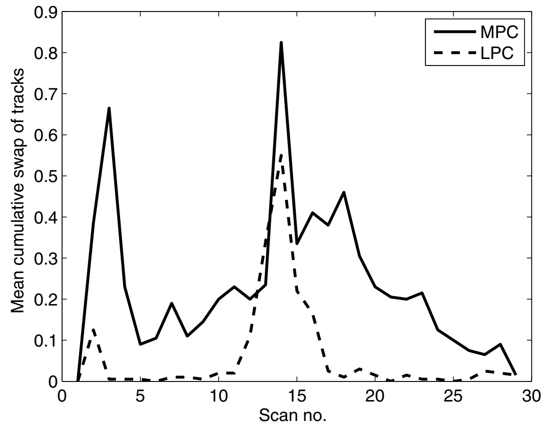

Figure 9: Mean cumulative swap of tracks, two target scenario, $\sigma = 1^o$.

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